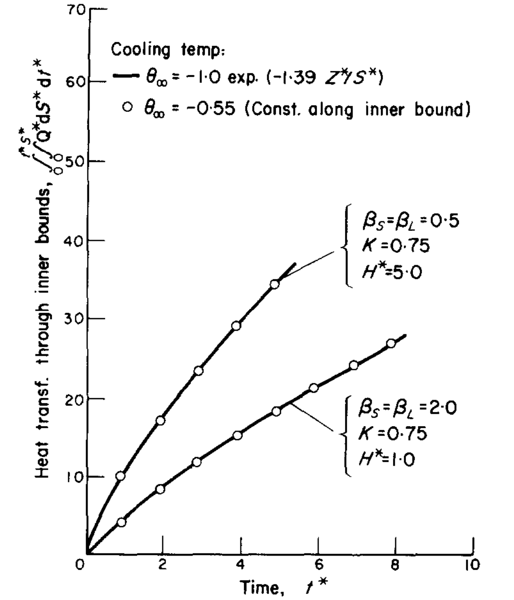

FIG. 4. Heat transfer through inner boundary surface of the cylinder calculated for axially varying temperature distribution (solid lines) and for constant "equivalent" ambient temperatures (circles). ($S^* = 10:1, r_0^* = 2:1, \theta_0 = 0$).

axial heat-transfer effects for various initial and boundary conditions are given in [1–3] and [6–10].

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HEAT FLOW ACROSS METALLIC JOINTS—THE CONSTRICTION ALLEVIATION FACTOR

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NOMENCLATURE

- T, temperature;
- Q, total heat flow;
- ΔT, temperature drop due to one side of constriction (see Fig. 1(a));
- R, thermal resistance of one side of constriction $\Delta T/Q$;
- k, thermal conductivity;
- a, radius of metallic contact spot (see Fig. 1(b));
- b, radius of cylindrical region feeding the contact;
- f, constriction alleviation factor, defined as the ratio of the actual resistance to that of the disc constriction resistance when bounded by a semi infinite conductor;
- u, t, dummy variables of integration.

INTRODUCTION

IN THE study of the thermal resistance of metallic contacts it is usual to model the conductors as a series of cylindrical elements concentric with the actual metal to metal contact spots, which are assumed to be uniformly distributed over the apparent zone of contact. The resistance to potential flow within each cylindrical element ("unit cell") consists of a bulk longitudinal resistance of the conductor and a constriction resistance caused by a pinching-in of the heat flow lines as they pass through the relatively small area of the actual contact spot. For most joints, these actual contacts are very widely spaced relative to their mean diameter, and in the absence of heat flow through a conducting fluid at the interface, the constriction resistance of each unit cell may be taken as that of a flat circular disc bounded on each side by a semi infinite conductor, viz. the classical "disc constriction resistance" to potential flow [1, 2]. However, when the flow fields of neighbouring spots interfere, this resistance must be modified by a "constriction alleviation factor" to account for the finite size of the unit cell. There have been several published forms of this factor based on simplifying assumptions [3, 4] or on empirical data [5]. It is the purpose of this brief note to present an analytical solution for the accurate evaluation of this factor and to compare this with published data.

It should be noted that the presence of a conducting fluid at the metallic interface can modify greatly the potential flow fields of the unit cells, and the reader is referred to more comprehensive treatments [3, 9] to deal with such cases. In general the contact resistance is lowered by the inclusion of a conducting fluid and also by increasing the number (or density) of actual contact spots within the joint.

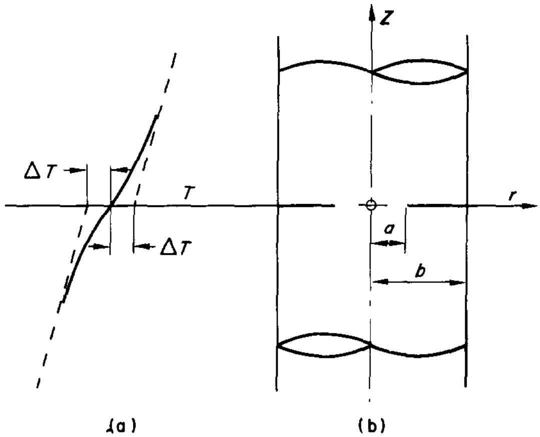

FIG. 1 Conditions satisfied by "T".

(i) $\nabla^2 T = 0$,
(ii) $\partial T/\partial z \to$ a constant as $z \to \infty$,
(iii) $\partial T/\partial r = 0$ at $r = b$,
(iv) $\partial T/\partial z = 0$ at $z = 0, a < r < b$,
(v) $T = 1$ at $z = 0, 0 < r < a$.

ANALYSIS

The parameters of the mixed boundary value problem are summarised in Fig. 1. Sneddon [6] considers a similar situation and, using cylindrical coordinates, chooses a solution for the potential field in the form

$$T(r, z) = C_0 Z + \sum_{n=1}^{\infty} \lambda_n^{-1} C_n J_0(r\lambda_n) \exp(-Z\lambda_n) \quad (1)$$

where λ_n are the roots of $J_1(r) = 0$.

This satisfies conditions (i), (ii) and (iii) of Fig. 1(b). Conditions (iv) and (v) lead to the dual series equations,

$$T(r, 0) = 1{\cdot}0 = \sum_{n=1}^{\infty} \lambda_n^{-1} C_n J_0(r\lambda_n) \quad \text{for} \quad 0 \leq r < a \quad (2)$$

and

$$\frac{\partial}{\partial z} T(r, z)\bigg|_{z=0} = 0 = C_0 - \sum_{n=1}^{\infty} C_n J_0(r\lambda_n) \quad \text{for} \quad a < r \leq b. \quad (3)$$

Representing

$$\frac{\partial}{\partial z} T(r, z)\bigg|_{z=0} \quad \text{by} \quad -\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\int_r^a \frac{t h(t)\,\mathrm{d}t}{\sqrt{(t^2 - r^2)}} \quad (4)$$

where $h(t)$ is an unknown function, and combining this with the formula for the coefficients of a Dini series [7], we obtain for

$$\left.\begin{aligned} C_0 &= 2\int_0^a h(t)\,\mathrm{d}t \\ \text{and} \quad C_n &= \frac{2}{J_0^2(\lambda_n)}\int^a h(t)\cos(t\lambda_n)\,\mathrm{d}t. \end{aligned}\right\} \quad (5)$$

Substitution of these into the dual series equations gives

$$h(t) - \int_0^a h(u)\,E_1(t, u)\,\mathrm{d}u = x(t) \quad (6)$$

where

$$x(t) = \frac{2}{\pi}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \frac{u T(u, 0)\,\mathrm{d}u}{\sqrt{(t^2 - u^2)}} \quad (7)$$

and where

$$E_1(t, u) = \frac{4}{\pi} + \frac{4}{\pi^2}\int_0^\infty \frac{K_1(y)}{y I_1(y)}[2I_1(y) - y\cosh(uy)\cosh(ty)]\,\mathrm{d}y. \quad (8)$$

Now C_0 is the terminal axial temperature gradient and the total heat flow Q is given by Fourier's Law as $\pi \,.\, b^2 \,.\, C_0 \,.\, k$. The classical disc constriction resistance is

$$R_c = \frac{1}{4ak},$$

hence

$$f = \frac{4a\,\Delta T}{\pi C_0 b^2}. \qquad (9)$$

If ΔT and b are each chosen as unit values,

$$f = \frac{2a}{\pi \int_0^a h(t)\,dt}. \qquad (10)$$

The factor f was evaluated, in digital form on the Monash University CDC 3200 computer, using steps of 0·02 for values of $a/b < 0{\cdot}20$, and in steps of 0·10 for $a/b > 0{\cdot}20$. The resulting values are shown as curve A in Fig. 2.

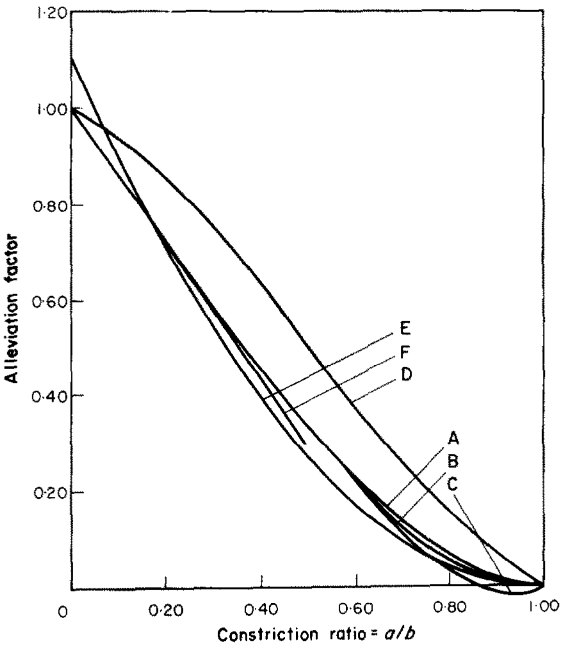

FIG. 2.

Key to curves:
A—Authors,
B—Roess (Appendix to [8], first 4 terms and compensating term),
C—Holm ([1] modified),
D—Cetinkale ([3] uncorrected),
E—Kouwenhoven–Sackett ([5] approx.),
F—Laming ([4] linear approx.).

DISCUSSION

In almost all practical situations involving heat flow across joints, the contact radius ratio a/b is low, probably less than 0·2. In this region of Fig. 2, the approximated values of the constriction alleviation factor as given by Holm [1], Laming [4], Roess [8], all match closely the values derived herein. The closest match is with the expression attributed to Roess referred to by Weills and Ryder [8], but the derivation of this cannot be traced. At very low values of the radius ratio, the poorest match is with that of Kouwenhoven and Sackett [5], believed to be derived from test results. At values of the radius ratio above about 0·08 the poorest match is with that of Cetinkale [3].

It is suggested that any of the previously published four analytical expressions for constriction alleviation factor may be acceptably accurate when used for predicting the resistance of solid contacts commonly encountered in engineering. The agreement amongst these expressions is much closer than the accuracy normally obtained from tests.

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